An Improved Evolutionary Algorithm for Solving Multi-Objective Crop Planning Models

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Abstract

In this paper, we analyse multi-objective optimization problems and provide useful insights about solutions that are generated using population based approaches. We formulate a crop planning problem as a multi-objective optimization model and solve two different versions of the problem using three different optimization approaches. The approaches considered are: ε-constrained method, a well-known multi-objective evolutionary algorithm NSGAII and our proposed multi-objective constrained algorithm (MCA). We compare the performance of our algorithm with other two algorithms and analyse the solutions from decision-making point of view. Our algorithm delivers superior solutions to nonlinear version of the crop planning model.

Keywords: Crop planning, multi-objective optimization, constrained problem, evolutionary algorithm

1.0 Introduction

Optimization techniques are widely used for solving complex practical problems in resource allocation, transportation and logistics, project selection, planning and scheduling. Although these problems are well-known in manufacturing and business sectors, there exist similar optimization problems in agricultural systems such as crop selection (Detlefsen and Jensen, 2004), country-wide crop planning (Sarker et al., 1997), irrigation planning (Raju and Kumar, 1999), vegetable production (Francisco and Ali, 2006), beef production (Mayer et al., 2001), wildlife and livestock management (Joubert et al., 2006), and sugarcane transportation (Higgins and Muchow, 2003). More optimization problems dealing with management of agricultural resources can be found in a recent survey paper by Weintraub and Romero (2006). As seen in these papers, the optimization problems were formulated as mathematical programming models and then solved using a variety of optimization methods. These models range from single to multi-objective, and from linear to nonlinear forms. The optimization methods used in the studies range from conventional methods to computational intelligence (CI) techniques such as genetic /evolutionary algorithms.

A single objective beef production model was solved using genetic algorithms and evolution strategies (Mayer et al., 1999 and 2001) and differential evolution (Mayer et al., 2005). A number of agricultural planning problems were formulated as multi-objective models (Joubert et al., 2006; Francisco and Ali, 2006; Raju and Kumar, 1999; Mainuddin et al., 1997; and Sarker and Quaddus, 2002). However these multi-objective problems were each solved as single objective model using either conventional goal programming or compromise programming approach. Although many decisions are dependent on such single objective solutions, approaches optimizing all the objectives simultaneously would provide superior solutions and better insights to the problem which indeed will help in better decision making in today's competitive market environment. Recently, deVoil et al. (2006) implemented a multi-objective evolutionary algorithm for solving a crop choice and sowing problem. In this paper, we have introduced a bi-objective linear crop planning model and later reformulated the model as a nonlinear program to incorporate some aspects of reality. Our intention is to provide some insight to the solutions of multi-objective optimization problems and demonstrate the use of two multi-objective optimization approaches for solving a crop planning problem.

We propose a modified evolutionary algorithm to solve the multi-objective mathematical models presented in this paper. Evolutionary algorithms have been used successfully to solve both single and multiple objective optimization problems, from the domain of operations research, in recent years (Sarker et al., 2003). There are many favourable reasons for choosing evolutionary algorithm in solving optimization problems. Consideration of convexity /concavity and continuity of functions are neither necessary nor needed to be known in evolutionary computation, however, these are a real concern in most mathematical programming techniques. EAs are a strong contender for problems with non-convex, discontinuous and multimodal functions. Thus we can avoid any simplification necessary for modelling complex problems. EAs are more suitable for multi-objective optimization because of their capability of simultaneous optimization of conflicting objective functions and generation of a number of alternative solutions in a single run. Due to the inherent parallelism, self-organization, adaptation and self-learning features of the EAs, they have been applied successfully to solve many problems where the classical approaches are either unavailable or generally lead to unsatisfactory results (Yao, 2001 and Patel et al., 2001). The detailed advantages of EAs over conventional optimization methods can be found in Sarker et al. (2003) and an excellent comprehensive review of evolutionary multi-objective optimization can be found in Coello (1999). As per literature, EAs are regarded as alternative optimization approaches for solving complex problems. Abido (2003), King and Rughooputh (2003) and Elliott et al. (2004) demonstrated the successful application of EAs in solving complex multi-objective optimization models. In addition, Abido (2003) and Elliott et al. (2004) reported that EAs outperformed classical methods in their cases.

Multi-objective optimization is a very important research topic in optimization and computing disciplines because of the multi-criteria nature of most real-world decision problems. There is no universally accepted definition of '*optimum*' in multi-objective problems as in single-objective optimization, which makes it difficult to even compare results of one method to another. The aim in a multiple objective optimization problem is to arrive at a set of *Pareto* optimal solutions. The Pareto solution points are also known as nondominated solutions in the sense that no other points would dominate them. The decision maker chooses a solution from the set of nondominated solutions based on his /her decision strategy. A solution $\mathbf{x}^* \in F$ is termed *Pareto* optimal if there does not exist another $\mathbf{x} \in F$, such that $f_i(\mathbf{x}) \leq f_i(\mathbf{x}^*)$ for all $i = 1,...,k$ objectives and $f_j(\mathbf{x}) < f_j(\mathbf{x}^*)$ for at least one *j*. Here, *F* denotes the feasible space (i.e., regions where the constraints are satisfied) and $f_j(\mathbf{x})$ denotes the j^{th} objective corresponding to the design $\mathbf{x}$. If the solution space is limited to *M* solutions instead of the entire *F*, the set of solutions are termed as nondominated solutions. Since in practice, all the solutions in *F* cannot be evaluated exhaustively, the goal of multiobjective optimization is to arrive at the set nondominated solutions with a hope that it is sufficiently close to the set of Pareto solutions. Diversity among these set of solutions is also a desirable feature as it means making a selection from a wider set of solution alternatives.

In this paper, we study the nature of multi-objective solutions produced by an evolutionary algorithm and compare them with two conventional multi-objective methods, Weighted Sum Method (WSM) and ε-constrained method, by solving a simple multi-objective test problem. This small problem helps to understand and interpret the results of a multi-objective problem. From there, we extend our study to crop planning problems to observe the behaviour in realistic problems. To solve the multi-objective crop planning problem, we have proposed a Multi-objective Constrained Algorithm (MCA) in this paper. Our proposed MCA is a modified and efficient version of an existing algorithm. The details of the algorithm are presented in a later section. Finally we solve two instances of the crop planning model and discuss our solutions.

The paper is organized as follows. After introduction, the crop planning problem is introduced and its mathematical model is discussed. In section 3, the features of three multi-objective methods are briefly described with a numerical example. In section 4, a new multi-objective constrained evolutionary algorithm is proposed for solving crop planning models. The results and discussions are provided in section 5. The conclusions and future research directions are summarized in the last section.

2.0 A Crop Planning Problem

In this section, we introduce a crop planning problem. Crop planning is related to many factors such as the type of lands, yield rate, weather conditions, availability of the agricultural inputs, crop demand, capital availability and the cost of production. Some of these factors are measurable and can be quantified. However, factors like rainfall, weather condition, flood, cyclone, and other natural calamities are difficult to predict. However, if the available information can be utilized properly, it may provide valuable suggestion despite the exclusion of nonquantifyable factors. The country, under consideration, grows a wide variety of crops in different seasons, and it has different types of lands. The types of lands we consider are single-, double- and triple-cropping land. The yield rate, the cost of production, and the return from crop are functions of soil characteristics (fertility and other soil factors), region, the crop being produced, cropping pattern and method (crop being produced and their sequence, irrigation, non-irrigation, etc.). For a single-cropped land, there are a number of alternative crops from which the crop to be cultivated in a year may be chosen. Similarly there are many different combinations of crops for double-cropped (two crops in a year) and triple-cropped (three crops in a year) lands. Different combinations give different outputs. The utilisation of land for appropriate crops is the key issue for the crop-planning problem in this paper. The problem is to provide an annual crop production plan that determines the area to be used for different crops while fulfilling the demand, land, capital, import and region limitations.

This looks like a well structured optimization problem. In the past, this problem was formulated as a single objective linear program and the corresponding model was then solved using a standard optimization solver (Sarker et al., 1997). The output of such a model would assist to plan annual crop harvesting which would maximize the return from a given area of land. This model can be designed either as a farm level or country wide crop planning. Sarker et al. (1997) implemented the model for a country wide planning where the agriculture development agency in the country is interested to see how the current practice differs from the optimal solutions. The agency has influence over the majority of individual farmers for crop selection as the agency provides loan to the framers, in terms of agricultural inputs and money. Beside the single objective optimal solution, the agency is also interested to see how the gross margin varies with the working capital to be distributed in a given year. In this section, the single objective model has been revised for better understanding and reformulated as a bi-objective model to incorporate working capital as the second objective function.

2.1 A Linear Crop Planning Model

Index:

i for a crop which can be considered for production
j a crop combination made up from i
k land type

Set:

CE set of crops that can be imported
CAL set of crops having area limitation
CIL set of crops having import limitation

Parameters:

n_1 number of alternative crops for single-cropped land
n_2 number of crop combinations for double-cropped land
n_3 number of crop combinations for triple-cropped land
$N1_j$ a crop in each j for single-cropped land, $j = 1, \ldots, n_1$
$N2_j$ the j-th crop pair of the possible crop combinations of double-cropped land, $j = 1, \ldots, n_2$
$N3_j$ the j-th crop triple of the possible crop combinations of triple-cropped land, $j = 1, \ldots, n_3$
YR_{ijk} yield rate that is the amount of production per unit area for crop i of crop combination j in land type k.
CP_{ijk} variable cost required per unit area for crop i of crop combination j in land type k.
P_i market price of crop i per metric ton
B_{ijk} gross margin that is the benefit that can be obtained per unit area of land from crop i of crop combination j in land type $k = (P_i * YR_{ijk} - CP_{ijk})$
IC_i gross margin from import of crop i (=Market revenue - Import cost)
D_i yearly demand of crop i
L_k available area of land type k
LT_k land type coefficient for land type k (=1, 1/2 or 1/3)
C_a working capital available, this indicates the total amount of money that can be used for covering variable costs.
A area suitable and available for crop i when $k = 1$
IL upper limit of total crop import

Variables

X_{ijk} Area of land to be cultivated for crop i of crop combination j in land type k.
I_i Amount of crop i that should be imported.

Objective function 1: The first objective is to maximize the total gross margin (from cultivated plus imported crops) that can be obtained from cropping in a single crop year.

$$\text{Maximize} \quad Z_1 = \sum_{j=1}^{n_1} \sum_{i \in N1_j} B_{ij(k=1)} X_{ij(k=1)} + \sum_{j=1}^{n_2} \sum_{i \in N2_j} B_{ij(k=2)} X_{ij(k=2)} + \sum_{j=1}^{n_3} \sum_{i \in N3_j} B_{ij(k=3)} X_{ij(k=3)} + \sum_{i \in CE} IC_i I_i \tag{1}$$

The first, second, third and fourth terms represent the gross margin from single crop land, double crop land, triple crop land and imported crop respectively. Note that there is only one crop for each j in single crop land, two crops in doubled crop land and three crops in triple crop land.

Objective function 2: The second objective is to minimize the total working capital required.

$$\text{Minimize} \quad Z_2 = \sum_{j=1}^{n_1} \sum_{i \in N1_j} CP_{ij(k=1)} X_{ij(k=1)} + \sum_{j=1}^{n_2} \sum_{i \in N2_j} CP_{ij(k=2)} X_{ij(k=2)} + \sum_{j=1}^{n_3} \sum_{i \in N3_j} CP_{ij(k=3)} X_{ij(k=3)} \tag{2}$$

Constraints:

Demand constraint: The sum of local production and the imported quantity of crop i in a year must be greater than or equal to the total requirements in the country.

$$\sum_j \sum_k YR_{ijk} X_{ijk} + I_{i \in CE} \geq D_i \qquad \forall i \tag{3}$$

Land constraint: The total land used for a given type of land must be less than or equal to the total available land of that type.

$$\sum_i \sum_j LT_k X_{ijk} \leq L_k \qquad \forall k \tag{4}$$

Here, for k = 1, 2 and 3, the coefficients (LT_k) are 1, ½ and 1/3 respectively. If a piece of land is used by two crops (in a double cropped land) one after another (consecutive production) in a given year, it is assumed equivalent to the use of half the land for one of these two crops in a year - that is LT_k= ½. This assumption makes the constraint (4) simpler and it is required only for land constraint.

Capital constraint: The total amount of money that can be spent for covering the variable costs in crop production must be less than or equal to the working capital available. Note that minimization of capital requirements is one of our two objectives formulated above. This additional constraint basically sets the upper bound of capital availability.

$$\sum_{j=1}^{n_1} \sum_{i \in N1_j} CP_{ij(k=1)} X_{ij(k=1)} + \sum_{j=1}^{n_2} \sum_{i \in N2_j} CP_{ij(k=2)} X_{ij(k=2)} + \sum_{j=1}^{n_3} \sum_{i \in N3_j} CP_{ij(k=3)} X_{ij(k=3)} \leq C_a \tag{5}$$

Contingent constraint: The area used for any crop under a crop combination for double- or triple-cropped land must be equal for every crop. For example, in a double-cropped land, the area used by two crops belonging to any crop combination must be equal.

$$X_{(i_1 \in N2_j)j(k=2)} - X_{(i_2 \in N2_j)j(k=2)} = 0 \qquad \forall j \tag{6}$$

In double cropped land, for a given crop combination j there is only two crops: i_1 and i_2 where i_1 is the first crop and i_2 is the second crop in the combination. Both crops use the same area of land but one after another. In a triple-cropped land, the area used by three crops belonging to any crop combination must be equal.

$$X_{(i_1 \in N3_j)j(k=3)} - X_{(i_2 \in N3_j)j(k=3)} = 0 \qquad \forall j \tag{7}$$

$$X_{(i_2 \in N3_j)j(k=3)} - X_{(i_3 \in N3_j)j(k=3)} = 0 \qquad \forall j \tag{8}$$

Here, i_1 is the first crop, i_2 is the second crop and i_3 is the third crop for combination j.

Area and import bound constraint: Due to soil characteristics and regional aspects, in some regions, the amount of area to be used for certain crops is restricted. For example, the unsuitability of certain lands for fruit cultivation needs to set an area limit for fruit. This is true only for single-cropped land. Similarly, a constraint needs to be set for import restriction as there is an upper limit on the importation of some crops.

Area bound: $$\sum_{i \in CAL} X_{ijk} \le A \qquad \forall j = 1, k = 1 \tag{9}$$

Import bound: $$\sum_{i \in CIL} I_i \le IL \tag{10}$$

Non-negativity constraint: The decision variables must be greater than or equal to zero.

$$X_{ijk} \ge 0 \qquad \forall i,j,k \quad \text{and} \qquad I_i \ge 0 \qquad \forall i \tag{11}$$

2.2 A Nonlinear Crop Planning Model

It is interesting to report here that, for a given crop, the yield rate in double- and triple- cropped land is little higher than the single-cropped land. This is due to frequent use of fertilizers and insecticides in double- and triple-cropped land. The difference is significant for triple-cropped land and a nonlinear relationship is established to reflect this change. We relate the change to the triple crop decision variables by expressing as $(x_{ijk})^b$. The value of *b* varies from situation to situation. For higher yield rate, the value of *b* will be more than one. In situations, where fertilizers are not appropriately used, the yield rate may decrease for double- and triple-cropped lands. So we assume the value of *b* will be less than one. In addition, the nonlinearity may arise due to soil characteristics and the level of agricultural inputs used. In our model, the triple crop variables for the first objective function and demand constraints will be assumed as nonlinear. As we all know, the functions generated from real world problems do not ensure nice mathematical properties as required by many optimization methodologies. The methodology proposed in this paper would able to handle any type of functions without analysing the complexity of mathematical functions.

3.0 Multi-objective Optimization Methods

In this study, to explain the multi-objective solutions, we used two conventional multi-objective optimization techniques widely used in practice and two evolutionary multi-objective algorithms. The conventional methods are: the weighted sum method (WSM) and ε-constrained method. We discuss these two methods briefly for completeness in this section. We also briefly discuss the nondominated sorting genetic algorithm (NSGAII), one of the most popular evolutionary methods, as our proposed method in this paper is a variant of NSGAII.

3.1 Weighted Sum Method (WSM)

The idea of this method is to associate each objective function with a weighting coefficient and minimize/maximize the weighted sum of the objectives (Coello, 1999). That means, the multiple objective functions are transformed into a single objective function and then solved as a single objective optimization problem. The modified problem can be represented as follows:

$$F(x) = \sum_{i=1}^{k} w_i f_i(\mathbf{x}) \tag{12}$$

Where $f_i(\mathbf{x})$ is the i^{th} objective function, *k* is the number of objectives, $w_i \ge 0$ and $\sum_{i=1}^{k} w_i = 1$. The weighting coefficients represent the relative importance of the objectives. The solutions may vary significantly as the weighting coefficients change. So WSM can generate a *trade-off* set of solutions by solving the modified problem using different weights. We used (LINGO, 2006) PC Release 10.0 for solving various problems based on WSM.

3.2 ε-Constraint Method

In this method, one of the objective functions is selected to be optimised and all other objective functions are converted into constraints by setting an upper bound to each of them (Coello, 1999). The problem to be solved is now of the form:

$$
\begin{aligned}
&\text{Minimize} \quad f_l(\mathbf{x}) \\
&\text{Subject to} \quad f_j(\mathbf{x}) \le \varepsilon_j \quad \text{for all } j = 1,....,k, \quad j \ne l \\
&\qquad \mathbf{x} \in S \\
&\quad \text{where } l \in \{1,\ldots,k\}.
\end{aligned} \tag{13}
$$

So the method is basically solving a single objective model each time. The optimal value of a single objective function *j* can be used as the upper limit of ε_j in this method. The level of ε_j is then altered to generate the entire Pareto optimal set. We used LINGO (LINGO, 2006) PC Release 10.0 for solving various problems based on ε-Constraint Method.

3.3 Nondominated Sorting Genetic Algorithm (NSGAII)

Srinivas and Deb (1994) proposed NSGA in 1994 for multi-objective optimization. The difference between the conventional single objective GA and NSGA-II lies with the assignment of fitness of an individual. There have been several improvements to the original algorithm and the latest form is referred as NSGAII (Deb et al., 2000 & 2002). The fitness of an individual in NSGAII is based on the non-domination level of an individual within a population size of *M*. Binary tournament selection, recombination, and mutation operators are used to create a child population of size *M*. Thereafter, the total population (*M* parents and *M* children) is sorted according to non-domination. The new parent population is formed by adding solutions from the first front and continuing to other fronts successively till the size exceeds the population size of *M*. The crowded comparison operator comes into play if the number of solutions at a particular non-domination level exceeds the number that can be accommodated in the new parent population. Diversity is preserved by the use of crowded comparison criterion in the tournament selection and in the phase of population reduction.

3.4 Interpreting Solutions of a Two Dimensional Multi-objective Model

In this section, we solve a simple mathematical model using an evolutionary algorithm, WSM and ε-constrained method to analyse the multi-objective solutions in both objective space and the solution space (also known as variable space), and compare the behaviour of these three methods. In a later section, we solve a practical crop planning problem using our own method to demonstrate the use of simultaneous multi-objective approach on realistic problems.

Let us consider a model with two objectives, two variables and two constraints:

$$\text{Maximize } Z1 = 2x_1 - x_2 \tag{14}$$

$$\text{Maximize } Z2 = -x_1 + 3x_2 \tag{15}$$

$$x_1 + x_2 \le 4 \tag{16}$$

$$x_1 + x_2 \ge 1 \tag{17}$$

$$0 \le x_1 \le 3, 0 \le x_2 \le 2 \tag{18}$$

The feasible solution space (shaded area) is presented in Figure 1. The directions of Z_1 and Z_2 increase are also shown. To apply the weighted sum method, we used the composite objective as w^*Z_1 + (1-w)*Z_2, where $0 \le w \le 1$. The plot of composite objective against weight *w* and the plot of Z_1 and Z_2 are shown in Figures 2(a) and 2(b) respectively. In Figure 2(a), the points represent the composite objective values for *w* = 0 to 1 with increment of 0.1. There are only four points on the Pareto frontier against 11 points in Figure 2(b).

For a given weight *w*, the composite objective makes the problem a single objective linear program. By changing the weights, we are basically changing the coefficients of the variables in the objective function. These changes may influence the solutions depending on the sensitivity and range of coefficients. In this problem, we used 11 uniformly distributed weights ranging from zero to one. In Figure 2, the links between the composite objective values and their corresponding solutions in the Pareto front and the solution space are shown. According to the corner point strategy in simplex method (a linear programming approach), if we change the number of weights from 11 to 101 (i.e., with an increment of 0.01 instead of 0.1) the number of solution points reported will still be four as the feasible solution space remains unchanged.

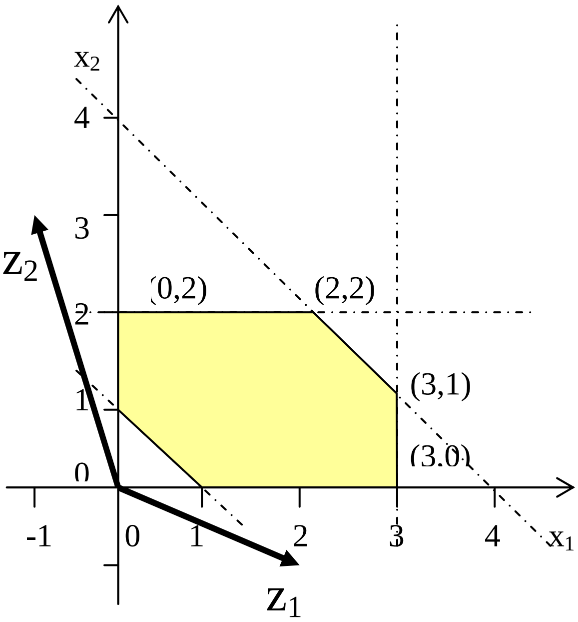
Figure 1:

Shaded area: feasible region

Bold arrows: direction of Z_1 and Z_2 increase

Vertices of the solution space: (1,0), (3.0), (3,1), (2,2), (0,2) and (0,1)

Single objective optimal solution: (0,2) for Z_2 and (3,0) for Z_1



We solved the above problem using NSGAII with a population size of 100 and allowed it to evolve over 100 generations. The other parameters for the run include a probability of crossover of 0.9, probability of mutation as 0.025, distribution index of crossover as 10 and the distribution index of the mutation operator as 20. Figure 3(a) represents the nondominated solutions, which correspond to the feasible solutions indicated in Figure 3(b) (three dark line segments). These feasible solutions include the four vertices produced by the weighted sum method as expected. As per the weighted sum method, each of four vertices represents a single objective optimal solution for a given linear combination of two objective functions. The number of points cannot be increased using this method as it will be restricted to the possible corner points within, and including, the two extreme single objective solutions.

Figure 2(a)

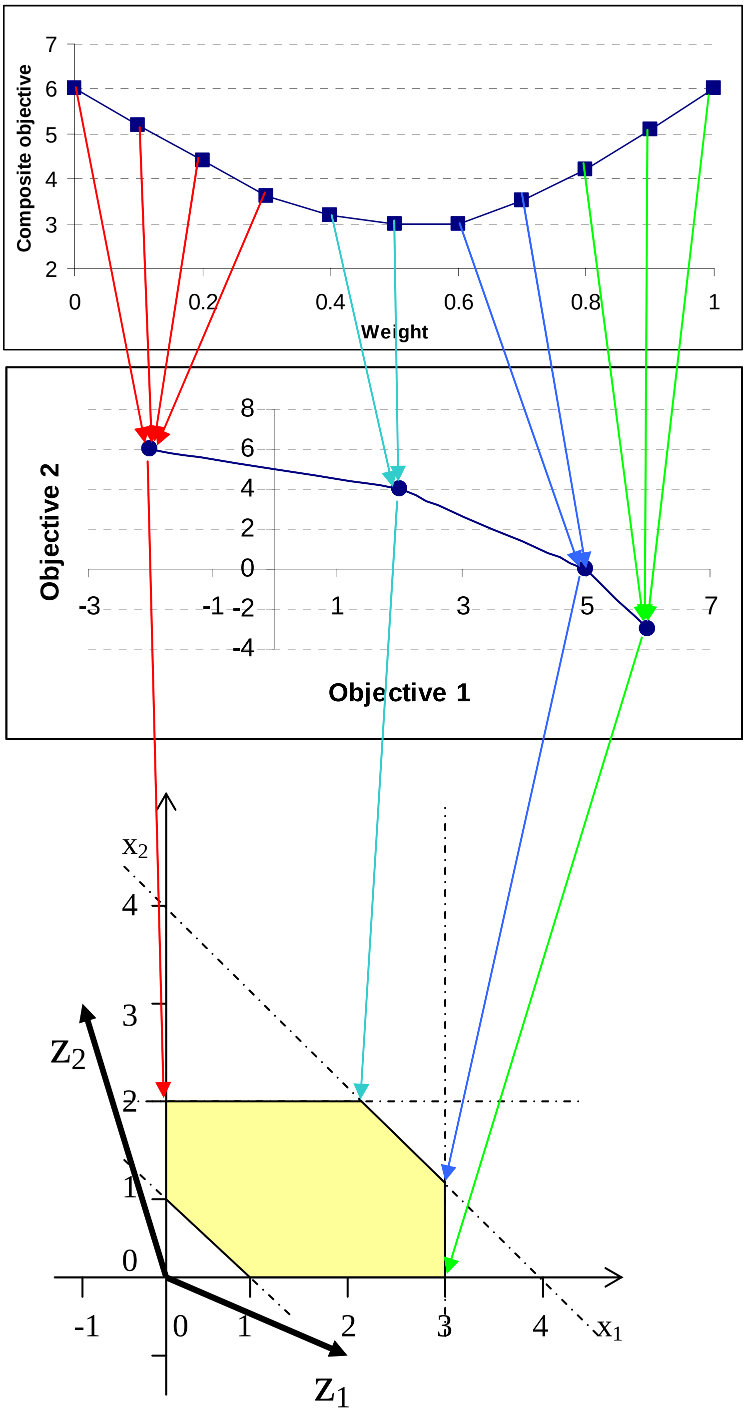
The left four points represent left most point in the Pareto frontier. The right three points respresent the right most point in Figure 2(b). Each of middle two points is for one point in the Pareto frontier.

Figure 2(b)

Shows how the points on the Pareto frontier correspond to their respective solutions in the feasible solution space.

Figure 2(c)

The solutions correspond to only four vertices: (3.0), (3,1), (2,2) and (0,2).



In order to obtain a nondominated set of points, one can systematically generate them using a ε-constraint method for either of the objectives (see Figure 3c). The bounds of objectives can be obtained by solving two single objective optimization problems and the ε limits can be generated depending upon the number of chosen intervals. We have used this procedure to generate and compare solutions with those obtained using NSGAII.

Figure 3(a)

The set of Nondominated solutions obtained using MEA.

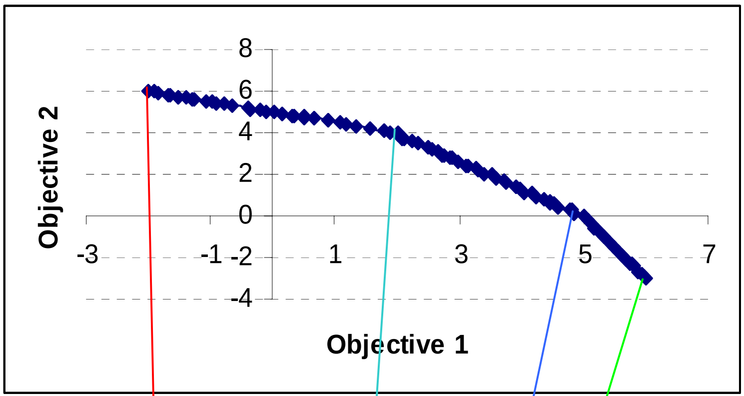


Figure 3(b)

The corresponding set of solutions from Figure 3(a) mapped to the variable space. The set of solutions lie on three lines with vertex (3, 0) to vertex (3,1), (3,1) to (2,2) and (2,2) to (0,2).

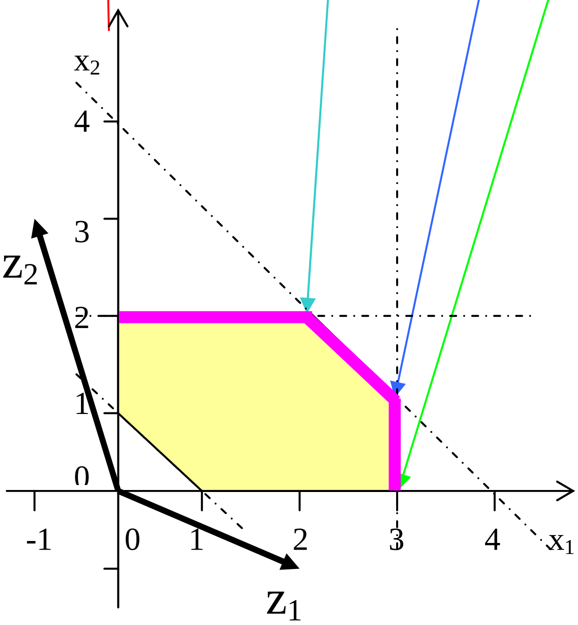
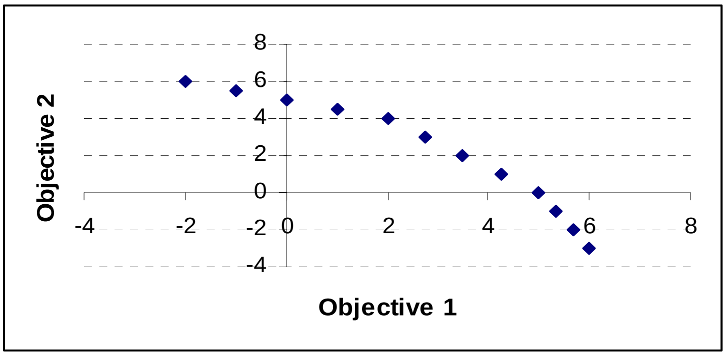


Figure 3(c)

The set of Nondominated solutions obtained using ε-constrained method.



As observed, NSGAII provides a good number of points on the nondominated front which means more alternatives to a decision maker to choose from. We must mention here that the evolutionary multi-objective algorithms generate a set of nondominated solutions in a single run. Once the problem has been solved, the decision makers usually choose a single point from the nondominated front (i.e., objective space solution as of Figure 3a) which suits his/her decision strategy. In this problem, the alternative solutions in the solution space (Figure 3b) form three line segments at the boundary of the feasible solution space. What do these alternative solutions really mean in practice? They can be interpreted as follows. They are simply the best possible feasible solutions (here optimal) for different combinations of objectives within the range of objectives' values. The ranges of objective values can be obtained easily from the single objective solutions. For a two-objective case, if we fix the value of one objective the value of other objective corresponds to the best possible feasible solution, which is optimal in single objective context, would be a candidate on the nondominated front. Note that the conventional methodologies (such as WSM and ε-method) can be used in generating Pareto frontier (as shown above) as long as the developed functions satisfy certain mathematical properties (such as differentiability and convexity /concavity). However, most practical problems are not driven by nice mathematical properties. So EAs are becoming a popular multi-objective optimization tool to the researchers and practitioners.

4.0 Solving the Crop Planning Model

As indicated by Sarker et al. (1997), there are more than 100 crops in the country under consideration. The decision makers are interested in an aggregate planning model to make high level planning decisions. Hence all the crops are divided into 10 major crop groups. These are Aus rice, B-Aman rice, T-Aman rice, Boro rice, Wheat, Jute, Rabi crop, Kharif crop, Fruits, and Drugs & Narcotics. The details of crops under each group can be found in Sarker et al., (1997). Interestingly, first five crop groups are cereal. The number of crop combinations identified for single-, double- and triple-cropped lands is 10 (=n_1), 17 (=n_2) and 6 (=n_3) respectively according to the present cropping pattern. Any of the 10 major groups/crops can be produced in a year in the single-cropped land. There are 17 pairs of crops that can be produced (one after another of the pair) in a year in double-cropped lands while 6 combinations (3 crops in each group, one after another in a year) in triple-cropped lands. In fact, the 17 pairs and the 6 triples of crops consist of the 10 crop groups listed above. The corresponding single objective linear programming model consists of 68 variables and 45 constraints. This linear programming model can be solved using standard optimization packages such as LINGO (LINGO, 2006), GAMS (GAMS, 2007) and ILOG (ILOG, 2007).

In this study, as discussed earlier, we consider two objectives: gross margin maximization and variable cultivation cost minimization. In addition, we apply variable /constraint reduction technique (Sarker and Newton, 2007) which reduces the model to 39 variables and 15 constraints. The parameters of the model are taken from Sarker et al. (1997). As the model is linear, the two-objective model can be solved using both WSM and ε–constrained method as discussed earlier. However, considering the ε–constrained method's capability in generating wide spread alternative solutions, we use this method for solving the multi-objective version of the crop planning model. The solutions produced by this method are presented in Figure 4. The unit of cost and gross margin is 'Taka' which is used as of Sarker et al. (1997). As per the current exchange rate, 1US$ is equal to 68 Taka.

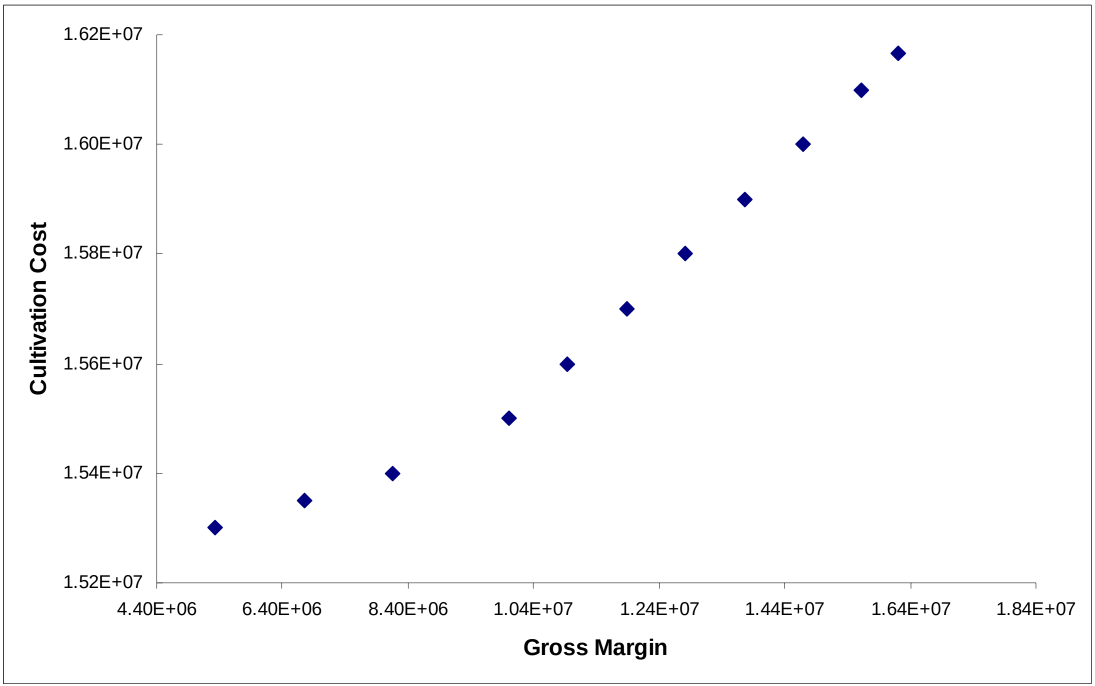


Figure 4: Pareto frontier obtained for a linear crop planning model using ε–constrained method

It is well known that any multiobjective evolutionary algorithm can generate a good number of alternative solutions in a single run, to build the Pareto frontier, irrespective of the properties of the objective functions and the solution space. On the other hand, EAs cannot perform better than conventional optimization method when solving linear programming based models. However, in a single run the conventional method, such as WSM and ε–constrained method, will produce one single point on the Pareto frontier. If we require many points on the Pareto frontier, it will take too much computational time and manual effort. Keeping this in mind, we tested NSGAII by solving our bi-objective linear programming model. It is interesting to observe that NSGAII is very sensitive to the initial population and it cannot find a single feasible solution using many different seeds as experimented. That motivates us to improve the performance of NSGAII by proposing a number of modifications. The modified algorithm is our proposed multi-objective constrained algorithms (MCA). We know once the algorithm is developed, the multi-objective problems with any function properties can be solved.

4.1 Multi-objective Constrained Algorithm (MCA)

We have developed an evolutionary algorithm to solve the above problem. This algorithm is a variant of NSGAII (Deb et al., 2000 and 2002) and has a major difference in the process of population reduction. In the process of population reduction from a size of 2M to M, the method insists on maintaining not only the end points of the objective space but also maximum and minimum values of the variables. The process is certainly a more computationally expensive than NSGAII and can be thought as a diversity maintaining mechanism which might be useful for problems where the diversity in the variable space is important. The benefits of maintaining diversity in both objective and variable space has been highlighted in Chan and Ray (2005).

The pseudo-code of the Multi-objective Constrained Algorithm (MCA) is provided below.

1. $t \leftarrow 0$
2. Generate M individuals representing a population: $Pop(t) = \{I_1, I_2, \ldots, I_M\}$ uniformly in the variable space.
3. Evaluate each individual: Compute their objectives and constraints i.e., $f_k(I_i)$ and $c_j(I_i)$; for $i = 1,2,\ldots,M$ individuals, $k = 1,\ldots,O$ objectives and $j = 1,\ldots,Q$ constraints.
4. Select two parents P1 and P2. (The procedure for selection is described below).
5. Create two children C1 and C2 via crossover and mutation of P1 and P2.
6. Repeat steps (4) and (5) until M children are created.
7. Evaluate M children.
8. Merge M parents and M Children to form a population of size 2M.
9. Rank 2M solutions
10. Retain better performing M solutions from the above 2M solutions.
11. $t \leftarrow t+1$
12. If $t < T_{max}$ then repeat steps (4) through (10), Else Stop.

T_{max} denotes the maximum number of generations.

4.1.1 Parent Selection

The procedure for selecting a parent P1 is described below and the same applies to selecting P2.

- 4.1 Select two individuals (R1 and R2) from the population of M solutions using a uniform random selection.
- 4.2 If R1 is feasible and R2 is infeasible: R1 is selected as the parent and vice versa.
- 4.3 If both R1 and R2 are infeasible: One which has the minimum value of the maximum violated constraint is selected as the parent.
- 4.4 If both R1 and R2 are feasible and R1 dominates R2: R1 is selected as parent and vice versa.
- 4.5 If both R1 and R2 are feasible and none dominates each other: A random selection is made between R1 and R2.

The points 4.1 – 4.5 discussed above are the detail description of Step 4 of our proposed algorithm MCA.

4.1.2 Crossover and Mutation

We have used simulated binary crossover (SBX) and the polynomial mutation for the real variables as adapted in NSGAII to create two children from a pair of parents in Step 5. As for integer variables, the real values are casted to integers.

4.1.3 Ranking

The procedure for rank computation is as follows:

9.1 Separate the set of 2*M* solutions to a set of feasible and a set of infeasible solutions.
9.2 Perform a nondominated sorting to assign ranks to the solutions in the feasible set.
9.3 Rank the solutions in the infeasible set based on their maximum value of a violated constraint.
9.4 Update the ranks of the solutions in the infeasible set by adding the rank of the worst feasible solution to each.

4.1.4 Retaining M Solutions

The procedure to retain M solutions from a set of 2*M* solutions is presented below:

10.1 Rank the set of 2*M* solutions. (The procedure for ranking is described below).
10.2 If the number of rank=1 solutions (i.e. nondominated solutions) is less than or equal to M, select top M solutions based on their rank and copy them to the new population.
10.3 If the number of rank=1 solutions is more than *M*, follow the following steps:

- 10.3.1 Select the solutions which have a minimum value (assuming minimization in all objectives) in any of the objectives and copy them to the new population.
- 10.3.2 For every variable, copy two solutions to the new population which has its minimum and the maximum value if they have not been copied yet (including step i).
- 10.3.3 For the remaining rank=1 solutions, the sequence of who goes in first to the new population is decided as follows:
  - 10.3.3.1 Compute the score by inserting the solution into the new population one at a time. The score is the minimum Euclidean distance computed between the solution attempting to enter with all other existing solutions in the new population based on the objective function space. Scaled values are used for the score computation i.e. the objective space is scaled using the maximum and minimum values in each dimension based on the set of rank=1 solutions.
  - 10.3.3.2 The solution with the highest score is allowed to go into the new population unless it has been copied earlier in which case the solution with the next score goes in.
  - 10.3.3.3 The steps (1) and (2) are repeated until the new population has a size of *M*.

4.1.5 Properties

The properties of the algorithm are:

(a) A feasible solution is always preferred over an infeasible solution. This is a commonly adopted practice, although one might argue that it's better to retain a marginally infeasible solution rather than a bad feasible solution.
(b) Step (i) in the above procedure ensures that the endpoints in the objective space are inserted into the new population and the extent of the nondominated front is preserved.
(c) Step (ii) is a means to maintain variable diversity i.e. to include a possibility of retaining variable values which might be useful.

5.0 Results and Discussion

In all our experiments we have used the same random number generator with the same seeds for MCA and NSGAII in order to ensure that the initial population is identical. The probability of crossover was set to 0.95 and the mutation probability was set to 0.025 (nearly 1/39 as there are 39 real variables). The distribution index of the crossover operator was set to 10 and for the mutation operator it was set to 20. All the cases were run with a population size of 100. The number of generations allowed was 1000 and 2000.

In this experiment, NSGAII struggled to find feasible solutions in too many runs. To quantify the failure rates of NSGAII, we carried out another set of experiments for a total of 128 runs with population size of 100 and 200, generation of 1000 and 2000, crossover probability of 0.9 and 0.95, mutation probability of 0.025 and 0.020, distribution index of mutation 10 and 20, and distribution index of crossover 10 and 20. In this experiment, we observed that NSGAII failed 88 (69%) times, to find feasible solution, out of 128 runs as opposed to 14 (11%) times for our algorithm.

To extend our testing beyond well behaved models as test cases for evolutionary algorithms, we have solved another instance of the problem which allows nonlinear relationship for triple-cropped land as discussed in section 2.2. In this instance, the variables for triple-crop were changed from linear to non-linear (by taking value of b = 1.1) in the first objective and the constraints where these variables are involved. The results are presented in Figure 5. For the nonlinear instance of the problem, it is interesting to observe that MCA reported better solutions as compared to both NSGAII and the ε–constrained method. We observed that this performance trend is consistent with different random seeds, that is, MCA located better solutions than the other two. It is appropriate to mention here that the ε–constrained solutions probably are the local optimum as LINGO (LINGO, 2006) does not guarantee global optimality for non-linear problems.

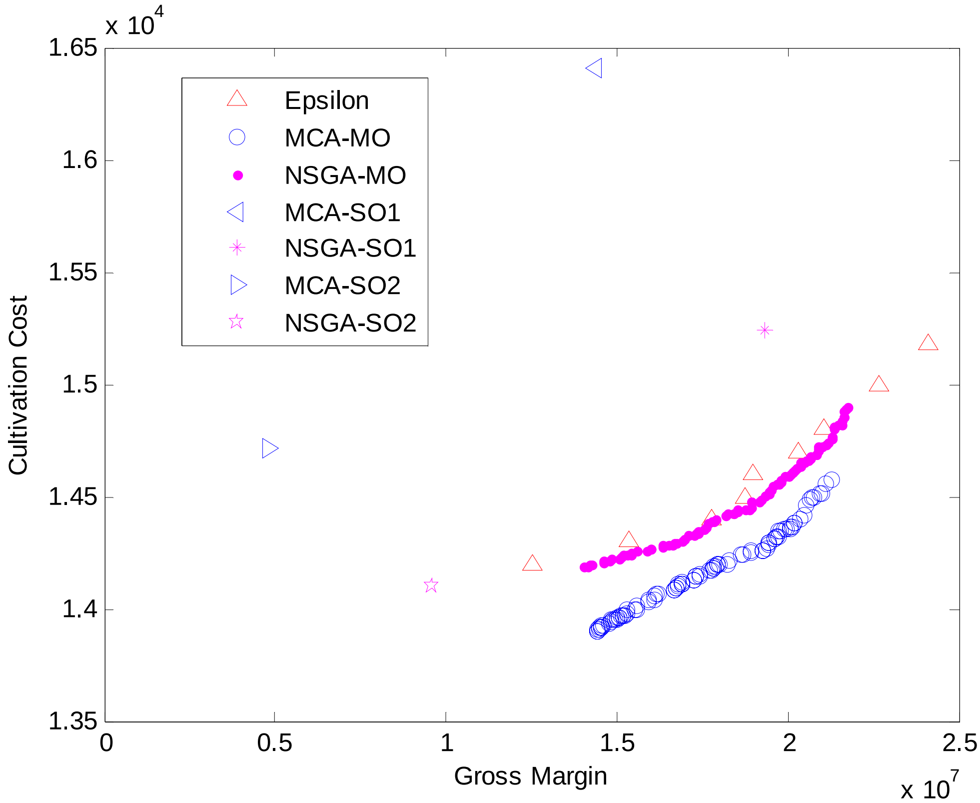


Figure 5: Results for the nonlinear crop planning model, for the ε–constrained method, MCA and NSGAII (MCA-SO1 and MCA-SO2 indicates the single objective solution with objective 1 and 2 respectively. Similar explanation is true for NSGA-SO1 and NSGA-SO2.)

6.0 Variable Diversity

Although generating the nondominated front as close to the Pareto front is the centre of attraction in multi-objective optimization, the solution space gets very little attention. However, the solution space is as important as the solutions in the objective space to many practitioners, as the resource requirements are different for different solution points. In this section, we investigate the variable diversity in the final population as obtained by MCA.

The Box plot for the nonlinear instance of the problem is presented in Figure 6. As we can see in this plot, one variable (X_{39}) has a wide spread, three variables have moderate spread, four variables minimal spread and the rest have nearly zero spread. Technically, whether a variable has a large spread or not is dependent on the gross margins (that is co-efficient) in the objective functions and the resource requirements (constraint functions). So only few variables are sensitive or contributing in generating the Pareto frontier which could be a useful information for the decision makers

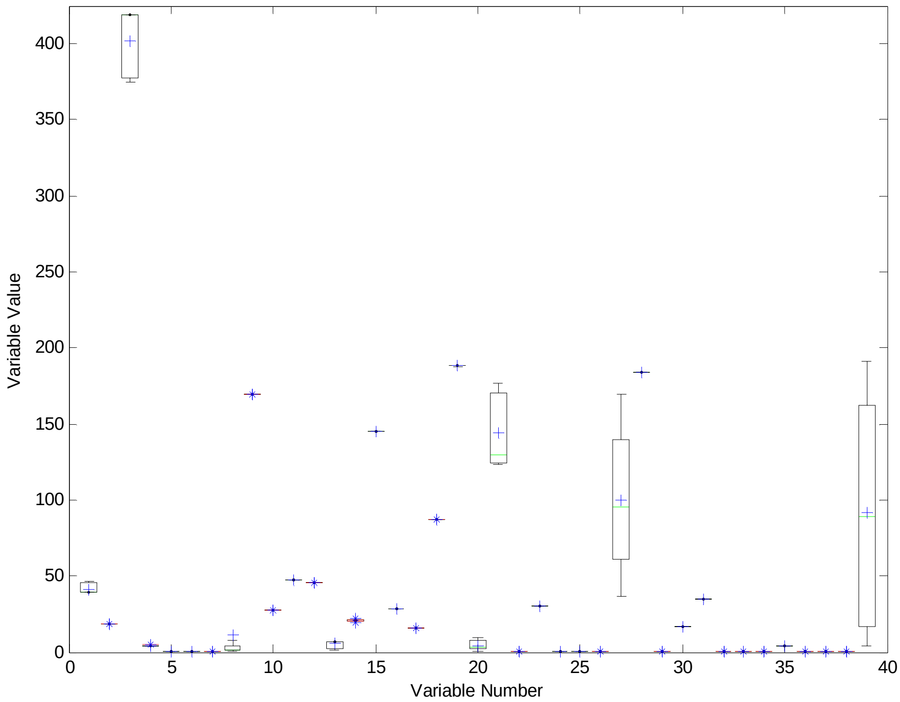


Figure 6: Box Plot of the variables in the final population as obtained using MCA for the nonlinear crop planning problem

7.0 Conclusions and Future Research

In this paper, we studied and interpreted the solutions of multi-objective optimization models by solving a simple linear constrained test problem using both conventional and simultaneous optimization techniques. We have also demonstrated the use of a genetic algorithm in solving multi-objective linear and nonlinear crop planning problem. It is interesting to report that for the linear multi-objective crop planning problem, NSGAII failed to find feasible solutions in 69% of the cases. As for the nonlinear crop planning problem, the Pareto solutions obtained using ε-constrained method was worse than both MCA and NSGAII. Our algorithm MCA performed better than NSGAII for both instances of the crop planning model.

Both instances of the crop planning problem considered in this paper have a single bounded feasible region. The constraints of these instances are in the forms of 'less than equal to' ($\leq$) and 'greater than equal to' ($\geq$). One might expect locating feasible solutions for these problems are easier as compared to the ones with equality constraints. However, the linear case is an instance where the evolutionary algorithms found it hard to locate feasible solution. Based on our experience, we believe these instances of the crop planning problem can be used as test problems for judging the performance of multi-objective constrained algorithms. Our solutions in form of data files will be made available through web for interested researchers.

As we discussed earlier, the variable diversity analysis may provide valuable information in accelerating the convergence of multi-objective algorithms. To make such an attempt, the diversity and range of variations can approximately be predicted from single objective solutions for using at the initial stage of the algorithm. We intend to carry out this investigation as one of our future research directions. In addition, the interactions of different variables at different segments of Pareto frontier can be analysed.

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